Piloting difficulties during the simulated flare manoeuvre are associated with height judgment, rather than the resolution of pitch attitude. If this effect is quantified, methods of compensation can be developed, and landing flare simulation will be much improved.

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Optimum Complementation of VOR/DME with Air Data

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Although the VOR/DME system has served efficiently as the primary domestic air navigation aid for some number of years, its navigational data are subject to sizable errors—errors produced principally by multipath propagation, which distorts the desired VOR signal. This paper demonstrates how augmenting the basic VOR/DME information with air data can significantly improve positional accuracy. The air data, when integrated, are used as a second source of position, and the data from the two sets of sensors are combined in an optimum filter. Results show that this optimally complemented system has an rms position accuracy 2.5 times better than that of unaided VOR/DME and that it can estimate wind components (for use in flight control) to about 20 knots rms. An example of the system's response to an actual noisy VOR signal is given.

Introduction

THE VOR/DME system† has been the primary domestic air navigation aid for a number of years. Its operational effectiveness is a matter of record, and the fact that it requires a minimum of relatively economical, easy-to-use, airborne equipment assures it continued longevity.

Although the system has proved to be quite efficient, its navigational data not infrequently contain sizable errors—errors springing primarily from multipath propagation phenomena, which distort the desired VOR signal. Because increasing air traffic densities demand increasingly accurate navigational information, the VOR/DME system errors loom larger and larger. Indeed, these errors may soon become a factor in limiting the capacity of airways.

Clearly, the key to more accurate and smoother guidance data lies in reducing VOR errors. Although it is true that improved system designs such as the doppler VOR[1] have become available, it is unlikely that such systems will be deployed in the near future because of the tremendous costs involved. Consequently for the near term any improvement in guidance data will have to be obtained through onboard signal processing. The most straightforward and most frequently used form of signal processing is low-pass filtering of the VOR output. Unfortunately, although low-passing quite effectively removes high-frequency errors, one finds that a filter designed to substantially attenuate frequencies on the order of 1 cycle/min begins to pose stability problems for the aircraft flight control system. Thus, the very errors that produce the most undesirable aircraft responses are difficult to extract by low-pass filtering.

Another form of signal processing is to augment the basic VOR/DME data with information derived from other sensors. A scheme of this type, wherein air data‡ are used to complement VOR/DME, is discussed in this paper. Specifically, the components of air data are integrated to provide an aircraft position measurement that is combined with the VOR/DME position information in an optimum filter. The performance of the resulting system is compared with that of the VOR/DME in an rms sense, and the response of the augmented system to some actual VOR data records is examined.

The optimally complemented system is shown to have the following characteristics: 1) Its rms position accuracy is substantially better than that of uncomplemented VOR/DME.

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† VOR (very high-frequency omnirange); DME (distance measuring equipment); the VOR and the DME measure bearing and distance, respectively, to a ground station.

‡ In this paper air data are defined to be true airspeed resolved into north and east components.

2) It can derive the wind vector with modest accuracy, a quantity which can be used for damping the flight control system and for computing accurate ground speed to assure precise ETA's. 3) It is quite insensitive to variations in both VOR and air data error parameters. 4) It does not tend to destabilize the flight control system because the navigation data used in control is not delayed.

Error Models

Any attempt to deal precisely with the design of a physical system requires a mathematical model. Here we want to improve VOR/DME performance by introducing air data. The technique for optimally combining these data sources depends upon the type and the magnitude of the errors in each source. Consequently, this section of the paper is devoted to developing reasonable error models for these sensors and to selecting appropriate numerical values for the parameters in the error models.

VOR/DME Errors

The geometry of VOR/DME position determination in the plane is shown in Fig. 1 where the station is at the origin of the coordinate system (x-east, y-north).

The aircraft has a bearing α and a range R with respect to the station so its position coordinates are

$$x = R \sin\alpha, \quad y = R \cos\alpha \tag{1}$$

The VOR measures α with an error $\Delta\alpha$, and the DME measures R with an error ΔR, so the measured coordinates are

$$\begin{aligned} x_m &= x + \Delta x = (R + \Delta R) \sin(\alpha + \Delta\alpha) \\ y_m &= y + \Delta y = (R + \Delta R) \cos(\alpha + \Delta\alpha) \end{aligned} \tag{2}$$

Assuming that $\Delta\alpha$ and ΔR are small, the errors, Δx and Δy, in the measured position coordinates are given to first order by

$$\begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} = \begin{bmatrix} R & \cos\alpha & \sin\alpha \\ -R & \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} \Delta\alpha \\ \Delta R \end{bmatrix} \tag{3}$$

A convenient measure of position accuracy is the rms radial error defined by

$$\sigma_R = [E(\Delta x^2 + \Delta y^2)]^{1/2}$$

where $E(\cdot)$ denotes mathematical expectation. Applying this definition to Eq. (3) we find

$$R = [R^2 \sigma_{\Delta\alpha}{}^2 + \sigma_{\Delta R}{}^2]^{1/2} \tag{4}$$

Here we have assumed that $\Delta\alpha$ and ΔR are independent, zero-mean stochastic processes with rms values $\sigma_{\Delta\alpha}$ and $\sigma_{\Delta R}$, respectively. Figure 2 is a plot of the radial error equation (4) for $\sigma_{\Delta\alpha} = 1.1°$ and $\sigma_{\Delta R} = 0.2$ naut miles, representative values for current equipment.

The straight line through the origin is the VOR contribution to radial error. We now see clearly that the VOR is the only significant contributor to radial error beyond ranges of about 20 miles. Consequently, the radial position error characteristic of uncomplemented VOR/DME is essentially a linear function of range, and its slope is the rms of the VOR error.

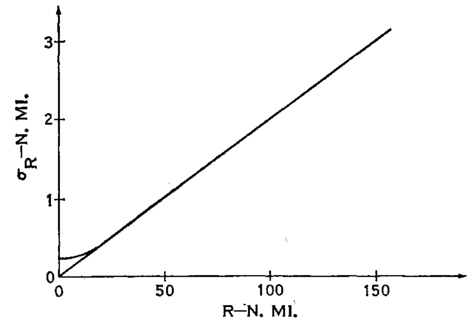

Fig. 2 VOR/DME radial position error.

In attempting to augment VOR/DME data, we must be aware of the frequency content, as well as the magnitude of the errors in these sensors. The frequency content of a stochastic process is contained in its autocorrelation function. Both the VOR and DME errors are assumed to be stationary and exponentially correlated, so we have

$$E[\Delta\alpha(t)\Delta\alpha(t + \tau)] = \sigma_{\Delta\alpha}{}^2 e^{-\beta}{}_{\Delta\alpha}{}^{|\tau|} \tag{5}$$

$$E[\Delta R(t)\Delta R(t + \tau)] = \sigma_{\Delta R}{}^2 e^{-\beta}{}_{\Delta R}{}^{|\tau|} \tag{6}$$

The exponential autocorrelation function is used because it is an economical first-order representation of a random process and because experience with these sensors indicates that it is a reasonable model for their errors. Examination of limited amounts of sensor data has indicated that the following correlation times are approximately correct: $1/\beta_{\Delta\alpha} = 10$ sec; $1/\beta_{\Delta R} = 300$ sec. The principal errors of the sensors have fixed spatial patterns, and it is the motion of the aircraft through these patterns that produces the time variation of the errors. Thus, the correlation times are velocity dependent, and the values given are for high subsonic speeds.

Air Data Errors

Air data as used in this paper consists of two basic measurements: the speed of the aircraft through the surrounding air mass and the direction of that motion. The first quantity is obtained from an airspeed sensor while the second is provided by a heading sensor. The measurements produce the vector $\mathbf{V}_{AM}$ which differs slightly from the true airspeed vector $\mathbf{V}_A$ in both magnitude and direction because of sensor errors. Figure 3 depicts the relationships among the various quantities which are involved. The aircraft ground-speed is its true airspeed plus wind which is the speed of the air mass. Thus, from Fig. 3, $\mathbf{V}_G = \mathbf{V}_A + \mathbf{V}_{WT}$ where $\mathbf{V}_{WT}$ is the wind vector. Also from Fig. 3,

$$\mathbf{V}_G = \mathbf{V}_{AM} + \mathbf{V}_W \tag{7}$$

where $\mathbf{V}_W$ is a vector which combines wind and the effects of

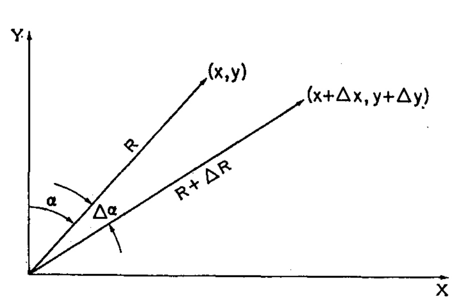

Fig. 1 VOR/DME relationships in the plane.

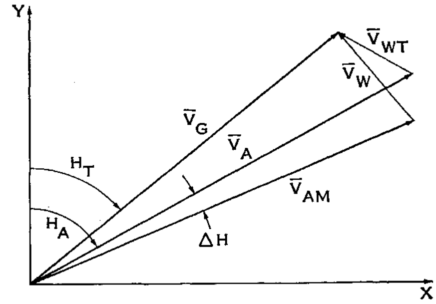

Fig. 3 Velocity relationships in the plane.

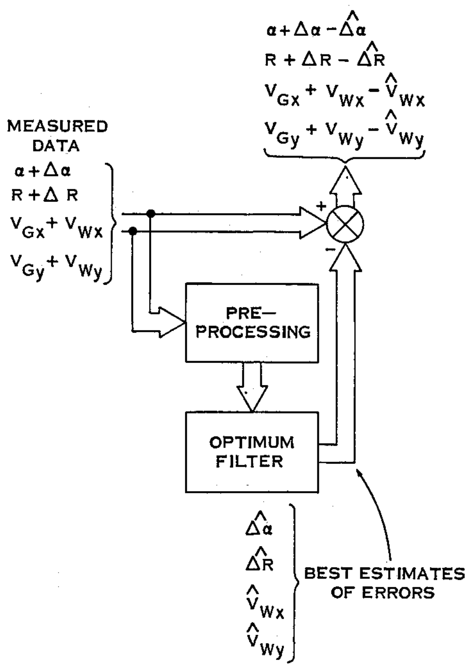

Fig. 4 System information flow.

errors in the airspeed and the heading sensors. Since, to an observer, the effects of sensor errors are indistinguishable from wind, let us call $\mathbf{V}_W$ the virtual wind vector.

The vector equation (7) can also be written as the following two scalar equations:

$$V_{Gx} = V_{AMx} + V_{wx}, \quad V_{Gy} = V_{AMy} + V_{wy} \tag{8}$$

Aircraft position in x-y coordinates is obtained by integrating the components V_{Gx} and V_{Gy}. Unfortunately the available measurement apparatus determines the components of $\mathbf{V}_{AM}$, not $\mathbf{V}_G$. However, the components of $\mathbf{V}_{AM}$ can be integrated and used as approximations $\hat{x}$ and $\hat{y}$, to the true position components x and y. Thus,

$$\int_0^t V_{AMx}(\tau)d\tau = \hat{x}(t) = x(t) + \Delta x(t)$$
$$\int_0^t V_{AMy}(\tau)d\tau = \hat{y}(t) = y(t) + \Delta y(t) \tag{9}$$

By employing the relations from Eq. (8) we conclude that the error components are

$$\Delta x(t) = -\int_0^t V_{wx}(\tau)dv, \quad \Delta y(t) = -\int_0^t V_{wy}(\tau)d\tau \tag{10}$$

Consequently, the errors in position derived from measured air data grow as the integrals of the virtual wind components. The largest contributor to the virtual wind is the true wind, since a good quality airspeed sensor has an error of only a few knots and a precision heading sensor typically will produce less than 10 knots error at high subsonic speeds. Therefore in the sequel we shall refer to the "wind," but it is to be understood that the quantity we are talking about also contains sensor errors.

The wind components are assumed to be uncorrelated, stationary, stochastic processes with exponential autocorrelation functions. High-frequency winds or gusts cause little or no net displacement of the aircraft. Therefore, we want to characterize the slow-varying, large magnitude winds which exist at cruise altitudes. Measured data describing this wind condition is essentially nonexistent. Thus, the selection of the rms value and the correlation time of the components is based upon engineering judgment supported by admittedly sketchy information. Nevertheless, the numerical values given below are thought to be realistic: $1/\beta = 10$ min; $\sigma = 40$ knots. The wind correlation time, as seen at the airpseed sensor, is a function of aircraft velocity, and the previous value is for high subsonic speeds. These parameters are used to describe both north and east wind components, and they will play an important role in the optimum filter.

If we evaluate the rms radial error of position obtained by integrating air data components, we find that this error is unbounded and grows proportionally to the square root of time.

Data Integration

In the preceding section, the nature of the position errors for both the VOR/DME and the air data system was discussed and mathematically described. Clearly, at any instant of time these systems provide two independent measures of aircraft position. We sense that perhaps some combination of the two is a better estimate of position than is either measure individually. This is indeed the case, and we shall see how the optimum filter is used to effect the best combination.

Position Differences

Let $\mathbf{P}_1$ and $\mathbf{P}_2$ be the position vectors simultaneously derived from VOR/DME and from air data, respectively, at any arbitrary time. Thus, $\mathbf{P}_1 = \mathbf{P} + \Delta\mathbf{P}_1$, $\mathbf{P}_2 = \mathbf{P} + \Delta\mathbf{P}_2$, where $\mathbf{P}$ is the true but unknown position while $\Delta\mathbf{P}_1$ and $\Delta\mathbf{P}_2$ are the error vectors in the two measured positions. Observe that the difference between $\mathbf{P}_1$ and $\mathbf{P}_2$ is independent of the true position; that is $\mathbf{P}_1 - \mathbf{P}_2 = \Delta\mathbf{P}_1 - \Delta\mathbf{P}_2$. This is an important observation because it tells us that the position differences are a function of sensor errors only and never depend upon the heading, the course, the present position, or the dynamic behavior of the aircraft.

Since $\Delta\mathbf{P}_1$ and $\Delta\mathbf{P}_2$ are the errors in VOR/DME and air data position, respectively, the position differences can be expressed in terms of sensor errors by employing Eqs. (3) and (10);

$$[\Delta\mathbf{P}_1 - \Delta\mathbf{P}_2] = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} =$$
$$\begin{bmatrix} R\cos\alpha & \sin\alpha & 1 & 0 \\ -R\sin\alpha & \cos\alpha & 0 & 1 \end{bmatrix} \begin{bmatrix} \Delta\alpha \\ \Delta R \\ X_W \\ Y_W \end{bmatrix} \tag{11}$$

where

$$X_W\,(t) = \int_0^t V_{Wx}(\tau)d\tau, \quad Y_W\,(t) = \int_0^t V_{Wy}(\tau)d\tau \tag{12}$$

Clearly, y_1 and y_2 are physically measurable, and they are linear functions of the quantities in the state vector at the right of Eq. (11). Under these conditions, linear filter theory tells us that, given the dynamic model describing the evolu-

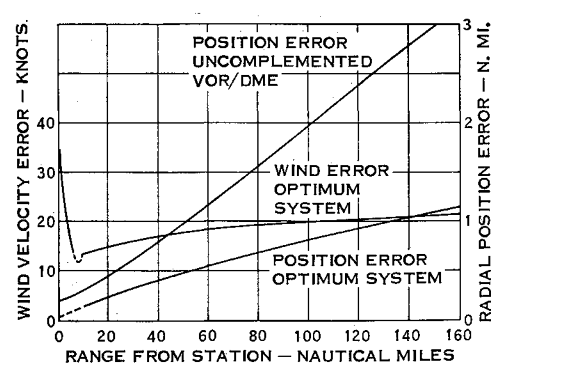

Fig. 5 Performance comparison: optimum system vs uncomplemented VOR/DME.

tion of the state vector, a filter which processes the measurables y_1 and y_2 can be constructed to provide state estimates which are optimum in the least squares sense.[2]

State Equation

The state equation which is needed for the optimum filter is implicit in the error descriptions given earlier. A stochastic process (stationary) whose autocorrelation is of the form

$$E[p(t)p(t+\tau)] = \sigma^2 e^{-\beta|\tau|}$$

satisfies the differential equation[3];

$$\dot{p} = -\beta p + w(t) \tag{13}$$

where $E[p_0^2] = \sigma^2$ and $E[w(t)W(t+\tau)] = 2\beta\sigma^2\delta(\tau)$, that is, $w(t)$ is white noise. Thus, the VOR error, the DME error, and the wind components all satisfy an equation of the form Eq. (13). Moreover, the integrated wind components given in Eq. (12) satisfy the trivial equations

$$\dot{x}_w(t) = V_{Wx}(t), \quad \dot{y}_w(t) = V_{Wy}(t) \tag{14}$$

Writing all the equations in vector form results in the state equation

$$\frac{d}{dt}\begin{bmatrix} x_w \\ y_w \\ V_{Wx} \\ V_{Wy} \\ \Delta\alpha \\ \Delta R \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -\beta_x & 0 & 0 & 0 \\ 0 & 0 & 0 & -\beta_y & 0 & 0 \\ 0 & 0 & 0 & 0 & -\beta_{\Delta\alpha} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\beta_{\Delta R} \end{bmatrix} \begin{bmatrix} x_w \\ y_w \\ V_{Wx} \\ V_{Wy} \\ \Delta\alpha \\ \Delta R \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ w_1 \\ w_2 \\ w_3 \\ w_4 \end{bmatrix} \tag{15}$$

All the parameters in this equation have been specified, and the covariance matrix of the white noise vector is fixed by the rms values of the error parameters. The output relation (11) can be rewritten to conform to the state equation (15) as follows:

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 & R\cos\alpha & \sin\alpha \\ 0 & 1 & 0 & 0 & -R\sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} x_w \\ y_w \\ V_{Wx} \\ V_{Wy} \\ \Delta\alpha \\ \Delta R \end{bmatrix} \tag{16}$$

In practice, a white noise vector is added to the right side of Eq. (16) to reflect the fact that no measurement can be made with perfect accuracy.

With the completion of the state equation and the output equation, all the information needed to construct the optimum filter is now available.

Table 1 System error parameters

Error source	Correlation time	rms value
VOR error	$1/\beta_{\Delta\alpha} = 10$ sec	$\sigma_{\Delta\alpha} = 1.15°$
DME error	$1/\beta_{\Delta R} = 300$ sec	$\sigma_{\Delta R} = 0.2$ naut miles
V_{wy}	$1/\beta_y = 600$ sec	$\sigma_y = 40$ knots
V_{wx}	$1/\beta_x = 600$ sec	$\sigma_x = 40$ knots

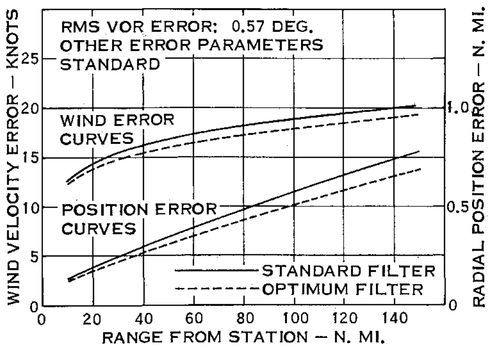

Fig. 6 Filter sensitivity to reduction in magnitude of VOR error.

Type of Mechanization

The optimum filter can be mechanized in either of two versions: one processes continuous data and requires solution of differential equations while the other processes discrete time data and depends upon solution of difference equations. Invariably, the latter is the more attractive because the airborne processor must be time-shared with other functions and because the measured data are likely to be provided in a sampled form. Consequently, we shall use the discrete filter for our analyses.

The filter does not generate directly the best estimates of aircraft position, but rather estimates the VOR and DME errors as well as the north and east components of wind. These estimated quantities must then be applied as corrections to the measured data as shown schematically in Fig. 4. The resultant data are the filtered or corrected version of the measurements, and it is optimum in the least squares sense.

A key feature of this optimally complemented VOR/DME system is the fact that it does not introduce delays which tend to destabilize the flight control system of the aircraft. One can see from Fig. 4 that the dynamic character of the measured data is unimpeded since the errors are computed in parallel with the major signal flow. Of course, the filter computations must be completed so rapidly that error parameters do not change significantly while estimates are being generated.

Performance Analysis

The preceding sections of this paper have developed sensor error models and shown how the sensor outputs can be com-

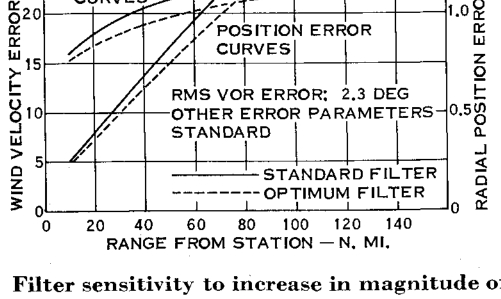

Fig. 7 Filter sensitivity to increase in magnitude of VOR error.

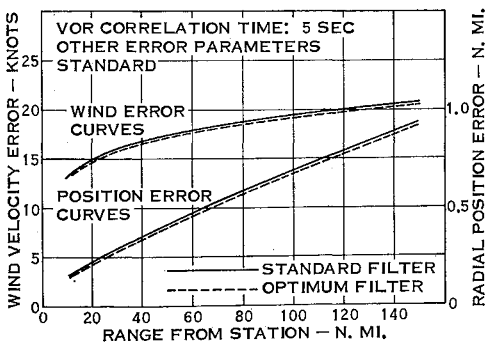

Fig. 8 Filter sensitivity to reduction in VOR correlation time.

bined in an optimum filter. Now we will examine the rms errors of the optimum system and compare its accuracy to that of uncomplemented VOR/DME data.

Optimum System

The analyses discussed herein were based upon the error source descriptions given in Table 1. The sampling interval of the filter was 5 sec. With the aforementioned error sources, more rapid sampling improves estimation accuracy very little.

Figure 5 shows the rms position error and the rms wind error of the optimally complemented system. For purposes of comparison, the position error of the unaugmented VOR/DME system is also plotted. The data show that the VOR/DME position error is approximately 2.5 times as great as that of the optimum system, and that the optimum system can estimate the wind components to roughly 20-knot accuracy.

Sensitivity Analysis

The optimally complemented system is optimum only when the statistical properties of the actual sensor errors match the nominal descriptions used in the filter design. We must be concerned with how the performance degrades when the filter sees inputs which differ from those expected. For example, in reality, the characteristics of VOR error may differ dramatically from station to station. Although the filter design is based upon a "typical" characteristic, it must perform acceptably well on all stations.

Figures 6–9 compare the performance of the filter based on nominal parameter values with that of the filter tailored to

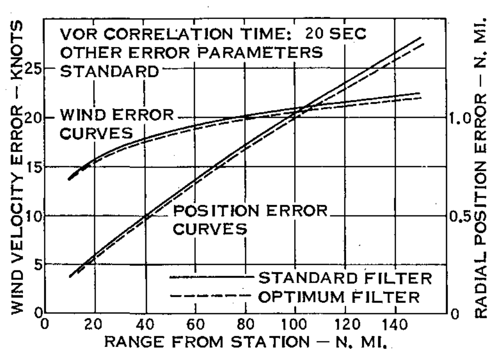

Fig. 9 Filter sensitivity to increase in VOR correlation time.

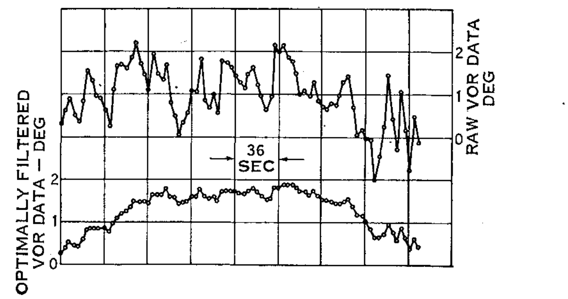

Fig. 10 Filter response to sample of VOR data.

each set of error conditions. The figures cover cases where the rms of the actual VOR error is one-half and twice the nominal value, and where the correlation time of the VOR error is one-half and twice the nominal value. For changes in error magnitude the nominal filter performance is within 10% of optimum. For variations in correlation time the maximum deviation from optimum is about 3%. Thus, it appears that the filter based on nominal parameter values is quite insensitive to rather wide variations in the VOR parameters.

Response to Actual VOR Data

The response of the optimum system to several records of actual VOR data was examined. To make the analysis as realistic as possible, random number generators were used to simulate the other system errors. Figure 10 shows the response to a typical record. Both the filtered and the unfiltered versions of the VOR record are given. The results clearly demonstrate that the optimum system sharply reduces the VOR noise, which for the record shown is in the range of 1–2 cycles/min.

Conclusions

This paper has demonstrated how air data can be used in an optimum manner for complementing VOR/DME information. The resulting system still depends on VOR/DME for primary navigational data, but utilizes air data to remove some of the short-term fluctuations due to VOR errors. The salient features of the system are 1) rms position accuracy is better than that of VOR/DME only by a factor of 2.5, 2) capacity to estimate the components of the wind with 20-knot accuracy, thus providing damping information for flight control and accurate ground speed information for computing ETA's, 3) insensitivity to variations in both VOR and air data error parameters, 4) it does not delay dynamic navigational data, and therefore has no destabilizing effect upon the aircraft flight control system.

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